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Research Article

A Bibliometric Analysis of Inverse Optimization Research: Trends, Impact, and Key Contributions

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ABSTRACT

Inverse optimization is defined as a process of reverse traditional mathematical optimization where parameters of an optimization model are determined such that given decisions will be either approximately or precisely optimal. The concept of inverse optimization was initially introduced and formulated in 2001. Research in this field has both theoretical and practical applications and has been applied in various areas. This study utilizes bibliometric analysis to determine the most impactful studies and researchers in the field of inverse optimization. Our research objective is to examine the current state of inverse optimization studies, based on data from the Web of Science database. Several classifications including leading journals, most frequently cited papers and influential authors are made. The results indicate that the majority of inverse optimization applications have been in the realm of cancer treatment and that the number of studies in this field has decreased during 2019-2020, that might be due to COVID-19 pandemic. Also, the USA and MIT (Massachusetts Institute of Technology) are among the most productive and influential entities in this field.

Keywords: Inverse Optimization, Bibliometric Analysis, Scientometry, Web of Science.

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INTRODUCTION

In recent decades, inverse optimization has garnered significant attention among researchers in the fields of mathematics and operations research. The objective of inverse optimization is to ascertain the parameters of an optimization model, often called the 'forward model.' This process aims to achieve a set of specified decisions that are either approximately or precisely optimal. This concept is further elucidated in Figure 1.

The initial application of inverse problems in geophysical sciences can be traced back to Tarantola's groundbreaking work in 1987.[2] Following this, Ahuja made a landmark contribution to the field in 2001, formally introducing the concept of inverse optimization. Ahuja's model, significant in its approach, aims to minimally alter a predefined cost vector to make a specific observed solution within a feasible space optimal. This research avenue has been explored across various domains, including transportation, healthcare, robust optimization and sustainability, among others. In these contexts, inverse optimization serves to craft decision-making models that render a known set of decisions as optimal or nearly optimal.[3]

The theoretical development of inverse optimization has evolved substantially over time. Traditional methods, as elucidated by Ahuja and others, primarily focused on parameter extraction that ensured a single solution's optimality. However, recognizing the challenge of achieving absolute optimality in complex scenarios, the field has seen a shift toward generalized models. Expanding the boundaries of the field, Chan introduced a general approach to tackle sub-optimal or infeasible solutions in linear programming, leveraging the concepts of strong duality and dual feasibility.[4] Troutt extended the inverse linear programming model to include multiple feasible solutions, employing the principle of decisional regret to guide the model's development.[5,6] Ghobadi addressed the uncertainties inherent in observed data through robust inverse optimization, creating models equipped to manage worst-case scenarios.[7] The advancement in methodologies continued with Saez, Keshavarz and Bertsimas utilizing Karush-Kuhn-Tucker (KKT) conditions, the introduction of slack variables and the application of variational inequalities, respectively, to further refine inverse optimization models for diverse data situations.[8-10]

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In a notable advancement, Aswani proposed an innovative loss function to quantify the discrepancy between measured and estimated data, emphasizing the joint estimation of optimization problems and their optimal solutions.[11] Chan's comprehensive framework for inverse linear optimization specifically focuses on cases with a single observed feasible solution, incorporating a fit metric to assess the precision of the inverse solution.[12]

To the best of our knowledge, none have examined the state of the field from a bibliometric perspective. Bibliometric analysis, increasingly popular across various scientific communities, is an essential tool for identifying patterns in scientific contributions.[13] In this study, we aim to offer a comprehensive overview of research in the field of inverse optimization through a bibliometric approach.

To achieve this goal, we use the Web of Science (WoS) database, which has been a significant repository of bibliometric data since its creation by the Thomson Reuters Institute of Scientific Information in 1960. Covering the period from 2010-2020, WoS is one of the largest databases of its kind, providing easy access to bibliometric data.[14] It collects a wide range of bibliographic information, citations and references from scientific publications in various disciplines, dating back to 1945.

This study makes a significant contribution to the literature by providing current insights into the state of inverse optimization, offering a detailed view of recent publications from a bibliometric perspective. This information will be invaluable for researchers, aiding in the improvement of future studies in the field. Additionally, we seek to visualize scholarly networks and developments in inverse optimization to identify potential areas for future research collaboration.

Section 2 presents the research aims and objectives. Section 3 describes the methodology of our study. The results are presented in Section 4. The discussion and concluding remarks are in Sections 5 and 6, respectively.

RESEARCH AIMS AND OBJECTIVES

This study aims to provide a comprehensive bibliometric analysis of the field of inverse optimization. Our goal is to dissect the academic landscape of this domain, identifying key trends, leading contributors and prominent areas of application. By examining the bibliometric data extracted from the Web of Science database, we intend to offer an overview of the field's developmental trajectory and significant research outputs. Our research objectives are as follows:

To Catalogue the Most Impactful Research and Researchers

Identify and highlight the most influential studies, authors and institutions in the field of inverse optimization, thereby recognizing the pivotal contributions to the domain.

To Analyze Trends in Research Themes and Applications

Examine the evolution of research themes within inverse optimization, particularly focusing on areas like cancer treatment and other prominent applications.

To Explore Collaborative Networks

Investigate the nature and extent of collaborative research in inverse optimization, including institutional partnerships.

Our research questions are listed below

What are the key trends in Inverse Optimization Research? Which topics have dominated the field?

Who are the leading contributors in the Field? Which authors, institutions and countries have been most influential in shaping the field?

What future directions can be derived from the current state of research? Based on current trends and gaps, what potential research areas appear promising?

By addressing these questions, this research seeks to offer a holistic view of the inverse optimization field, shedding light on its historical progression, current state and potential future trajectory.

Bibliometric Method

The methodology used in bibliometric analysis and data visualization helps researchers to understand different aspects of studies in a field. Machine learning, big data algorithms and ANOVA are some of the approaches used in bibliometric analysis. In some studies like[15] and[16] ANOVA was used. Some others like[17-21] utilized big data and machine learning or deep learning, respectively. Many studies have also explored the use of big data, such as[22-24] Additionally, bibliometric and social network analysis has been applied in.[25]

To the best of our knowledge, there has not been a study in inverse optimization that has used these analysis methods. This study will provide valuable insights and contribute to the literature by using bibliometric analysis to assess the current state of inverse optimization research.

The research process of the present study involves the following steps

Data Collection

782 records are extracted from the WoS database using the search query "Inverse Optimization" in the title or abstract without any time period limitations. These records were analyzed based on annual publications, the publication of the most productive countries, institutions, journals and authors. The data were extracted from the WoS database.

Bibliometric Analysis

The descriptive bibliometric analysis was conducted to determine the most effective studies and researchers in the field of interest. Using VOSviewer and Microsoft Excel the strongest collaborative relationships were identified. These tools also used to depict co-citation, co-authorship, co-occurrence and bibliographic coupling networks of countries, institutions, journals, authors and papers.

Conclusions on Academic Research Trend

This part of the analysis reveals the key trends and patterns in the field of Inverse Optimization, identifying the most influential contributors and the evolution of research themes over time.

RESULTS

The subsequent section details the results obtained through the utilization of VOSviewer, a tool for constructing and visualizing bibliometric networks, in conjunction with data sourced from WoS database. This combination allows for an in-depth analysis and visual representation of the research trends, patterns and key contributors in the field of inverse optimization.

Publications Overview

The number of publications is a key metric for gauging the development trend of scientific research.[26] Figure 2 illustrates the types of records in our dataset, revealing an overall upward trajectory in the volume of papers published in the field of Inverse Optimization. However, this growth trend seems to have decelerated post-2018, with an exception in 2021. This observation is consistent with the global decline in non-COVID-related publications during the COVID-19 pandemic.[27] Furthermore, Figure 2 provides an analysis of the distribution of data by publication types, encompassing articles, proceeding papers, editorial materials, meeting abstracts and reviews within the realm of Inverse Optimization.

Overview of the Keywords

A keyword analysis of the titles and abstracts in our dataset was conducted to identify prevalent research themes in inverse optimization. Figure 3 depicts the network structure of keywords used in these publications, with Figure 3 highlighting that the size of the nodes correlates with keyword usage frequency. Notably, the three most frequent keywords used alongside 'Inverse Optimization' are 'optimization,' 'algorithm,' and 'radiation therapy.' Moreover, the top five specific keywords, excluding

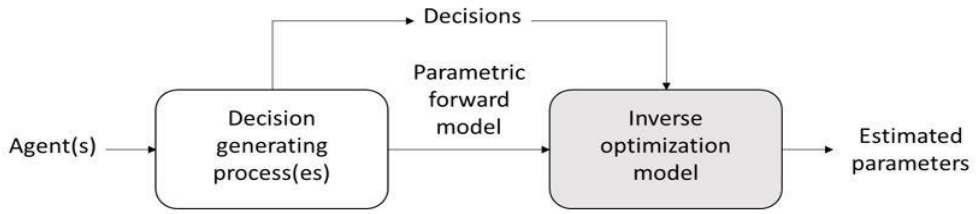

Figure 1: Methodology of Inverse Optimization.[1]

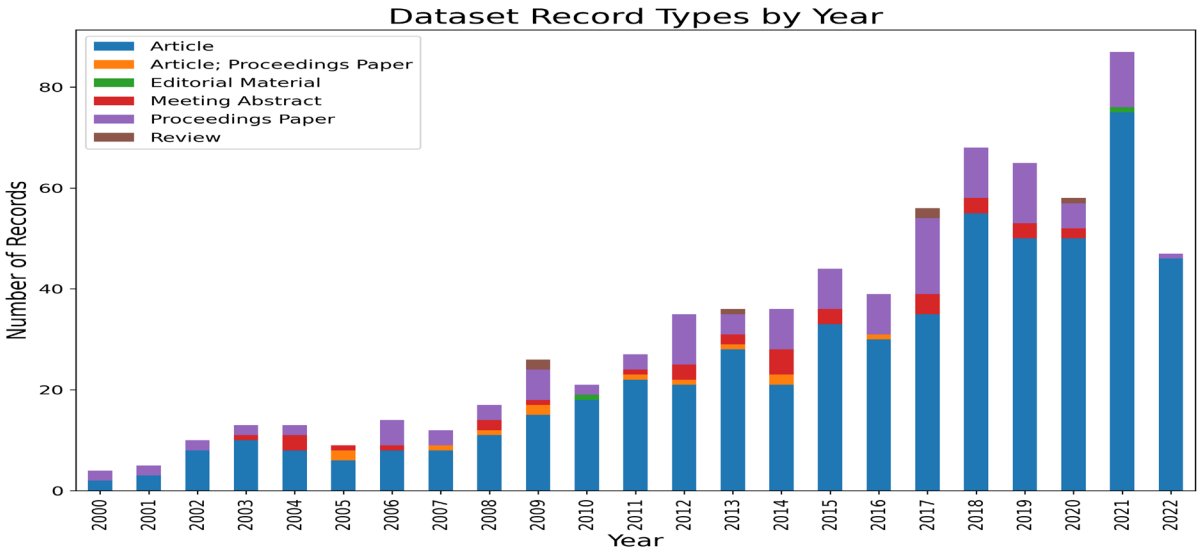

Figure 2: Annual distribution of publications on "Inverse Optimization" in the WoS database.

more general terms, include 'radiation therapy,' 'brachytherapy,' 'cancer,' 'IMRT" (Intensity-Modulated Radiation Therapy) and 'combinatorial optimization.' This suggests a significant focus of inverse optimization within the healthcare sector, particularly in cancer treatment. Additionally, the keyword 'inverse problem' appears frequently, reflecting its conceptual overlap yet distinctness from inverse optimization.

As outlined in study,[1] inverse optimization falls within the broader category of 'inverse problems,' which utilize data on the outcomes of a process to infer the causative factors.[28,29] However, inverse optimization is distinct in its approach, focusing on decision-making problems modeled through mathematical programming and utilizing solutions to these problems as data. Consequently, the methodologies and applications of inverse optimization diverge from the general field of inverse problems.

In Figure 3, as per the VOSviewer manual, the link strength denotes the co-occurrence relationship between two keywords. The thickness of each line reflects this strength, with thicker lines indicating a more robust connection between keywords. Furthermore, the size of a node in the graph correlates directly with the frequency of the keyword's occurrence; larger nodes represent a higher occurrence. Clusters also indicate potential for interdisciplinary collaboration. For instance, if a cluster connects keywords from both healthcare and mathematical modeling, it suggests opportunities for collaboration between these fields.

The analysis of the co-occurring terms with "Inverse Optimization" in Figure 3 predominantly highlights its significant application in healthcare, particularly in cancer treatment through radiation therapy and brachytherapy. Frequent references to "Optimization," "Algorithm," and "Combinatorial Optimization", 134, 63 and 43, respectively, underscore the essential role of algorithmic and optimization methods in this domain. The presence of specific cancer-related terms like "IMRT" and "Prostate Cancer," along with methodological terms like "Model" and "Simulation," reflects the field's interdisciplinary nature and its focus on improving treatment quality and planning. The strong linkages between these terms suggest concentrated research activity and potential collaborative efforts within healthcare optimization, especially in oncology. Also, keywords clustered around "inverse optimization" and "algorithm" represents a research area focusing on the development of algorithms specific to inverse optimization problems.

Overview of the Countries

Table 1 presents the data on countries with the highest frequency of publications in inverse optimization, showcasing both the number of articles and their citation counts for the top 10 countries. As detailed in Table 1, the United States emerges as the leader in publication frequency, followed by China and Canada in second and third positions, respectively.

Additionally, we have introduced an index measuring citations per document. According to Table 1, Austria, Spain and England rank highest on this index, indicating their respective impacts in terms of citation frequency relative to their document count.

The data from Table 1 on leading nations in inverse optimization research suggests several implications. The USA is the clear leader, with the highest number of documents, citations and total link strength, indicating its dominant role in the field. China follows, with a significant presence but lower impact in terms of citations per document. Canada, France and Germany also show strong

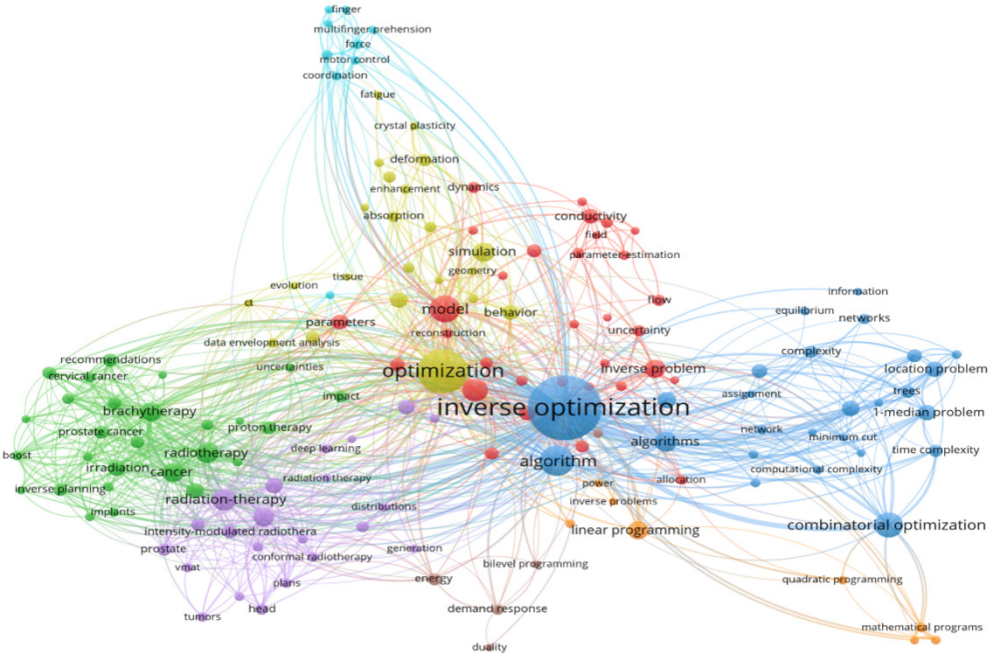

Figure 3: Term intersection diagram in inverse optimization domain.

contributions, with notable citation rates that suggest quality research output. Interestingly, Austria, despite a smaller number of documents, shows a high citation per document ratio, indicating highly impactful research. Similarly, Spain and England, while having fewer documents, show high citations rates, pointing to influential contributions in the field. The presence of countries like Belgium and South Korea highlights the global interest and diverse geographical representation in inverse optimization research. Overall, these statistics reveal a field with substantial and varied contributions from several key nations, reflecting both the quantity and quality of research across the globe.

The countries with the highest scientific production often have the highest scientific impact because a larger volume of documents generally leads to a greater likelihood of citations, reflecting a broader influence in the scientific community. However, Spain and Austria stand out for their high citation/document ratio, indicating that although they produce fewer documents compared to leading countries like the USA, their research has a significantly high impact per document. This could be due to the focused quality of research, specific niches of expertise, or higher collaboration rates with internationally renowned researchers, which enhances the visibility and citation rate of their publications. In contrast, countries with higher overall citations might benefit from a larger scientific community and a wider range of research areas, contributing to their total citation count despite a lower citation/document ratio.

Overview of the Institutions

Table 2 presents the institutions that published articles, along with the number of citations received by each institution and the most frequently used author keywords (For the table of the top 30 institutions, see Appendix A.) As evident from Table 2 and Table 5 (Appendix A.), the institution with the highest strength of link is the University of Toronto. The Massachusetts Institute of Technology (MIT) received the most citations (683). It is important to note that the number of citations is heavily influenced by the quality and productivity of scientific publications.

Also, common topics are observed among the institutions. For example, both Massachusetts General Hospital and Emory University share several common keywords, including IMPT, ADMM, Knowledge-Based Planning, etc. Upon reviewing related articles, we have observed collaboration between them.[30-33]

The implications of Table 2 and Table 5 (Appendix A.), highlighting the institutions with the highest overall link strength in inverse optimization research, are multifaceted. The University of Toronto, leading in link strength, suggests a central role in fostering collaborative research networks. MIT's highest citation count reflects its significant impact and prestige in the field. Notably, the Sahand University of Technology, with the most documents (282, see Appendix A.), indicates a strong focus on inverse optimization research, possibly signifying a specialized or emerging research area within the institution. The high citations per document ratio for the University of Florida (50.33), Graz University of Technology (47.8) and MIT demonstrate not only prolific research output but also high-quality and influential contributions to the field. These tables reveal a diverse set of institutions varying in their research focus, be it in terms of quantity, quality, or collaborative influences. Such diversity underscores the multidisciplinary and global nature of inverse optimization research, highlighting institutions that are key nodes in this expanding research network. These institutions, from different parts of the world, reflect the global interest and varied perspectives contributing to advancements in the field.

Leading Journals in Inverse Optimization

To provide a comprehensive overview of the most influential journals in the field of inverse optimization, we have ranked the top ten journals based on their number of publications. Although the number of publications may not necessarily reflect the quality of a journal, it can provide an indication of the status and prominence of journals in this field.

According to our ranking, the top three journals in the field of inverse optimization are Medical Physics, Physics in Medicine and Biology and the European Journal of Operational Research

Table 1: Leading nations with strongest total linkages among countries.

Country	Documents	Citations	Citations/Documents	Total link strength
USA	283	4264	15.07	108
Peoples R China	156	1262	8.09	62
Canada	58	744	12.83	35
France	49	563	11.49	33
Germany	49	649	13.24	30
Belgium	19	252	13.26	26
Austria	20	610	30.50	22
South Korea	17	163	9.59	17
England	23	398	17.30	16
Spain	17	372	21.88	16

Table 2: Institutions with the highest overall link strength.

Organization	Documents	Citations	Citations/ Documents	Total link strength	Author Keywords
University of Toronto.	21	524	24.95	11	Inverse optimization; Linear Programming; Treatment Planning; Machine Learning; Knowledge-Based Planning; Radiation Therapy; Clinical Pathway Concordance; Cancer Therapy (e.g., colon cancer, breast cancer, prostate cancer); Multiagent System; Network Analysis.
Massachusetts General Hospital.	7	50	7.14	10	Proton Therapy; Pencil Beam Scanning (PBS); Intensity Modulated Proton Therapy (IMPT); Alternating Direction Method of Multipliers (ADMM); Knowledge-Based Planning; Optimization; Inverse Optimization.
Emory University	11	77	7.00	9	Proton Therapy; Pencil Beam Scanning (PBS); Intensity Modulated Proton Therapy (IMPT); Alternating Direction Method of Multipliers (ADMM); Knowledge-Based Planning; Optimization; Inverse Optimization.
Harvard Medical School	6	52	8.67	9	Proton Therapy; Pencil Beam Scanning (PBS); Intensity Modulated Proton Therapy (IMPT); Alternating Direction Method of Multipliers (ADMM); Minimum Monitor Unit (MU); Knowledge-Based Planning; Inverse Treatment Planning; Brachytherapy.
MIT	15	683	45.53	8	Inverse Optimization; Robust Optimization; Linear Programming; Market Elements (Electricity Pool, Marginal Prices, Market Clearing); Mechanism Design; Auctions (Multiattribute Auctions); Equilibrium; Estimation (Nonparametric Estimation, Utility Estimation); Traffic Assignment; Flow Problems (Maximum-Flow Problem, Minimum-Cut Problem, Minimum-Cost Flow Problem, Minimum Mean-Cycle Problem); Combinatorial Optimization; Computational Complexity; Mixed-Integer Programming.

(EJOR) (as depicted in Figure 4). As previously discussed, a majority of papers in inverse optimization tend to focus on cancer treatment and this trend is reflected in our journal ranking as well. Among the top three journals, EJOR boasts the highest impact factor, having received a significant number of citations. In addition to EJOR, other journals included in the ranking, such as Operations Research Letters, Operations Research and Journal of Global Optimization, focus on operations research field while other journals focus on fields such as cancer treatment, radiation therapy and healthcare.

Leading journals are shaping the field of inverse optimization by serving as pivotal platforms for interdisciplinary collaboration, particularly between operational research and healthcare. In this regard, we can see those journals such as Brachytherapy, Journal of Applied Clinical Medical Physics, International Journal of Radiation Oncology Biology Physics, Radiation Oncology, Physics in Medicine and Biology and Medical Physics can all be categorized within the medical field. With top-tier journals like Medical Physics and Physics in Medicine and Biology focusing on the medical applications of physics, particularly in cancer treatment, they steer research toward practical and impactful healthcare solutions. Meanwhile, the European Journal of Operational Research and similar publications emphasize the theoretical and methodological rigor, ensuring that the advancements in the field are both scientifically robust and applicable. The frequency of publications in these journals reflects active research directions, with high citation rates underscoring the quality and influence of the research disseminated, thereby setting the tone for current and future research priorities within the field.

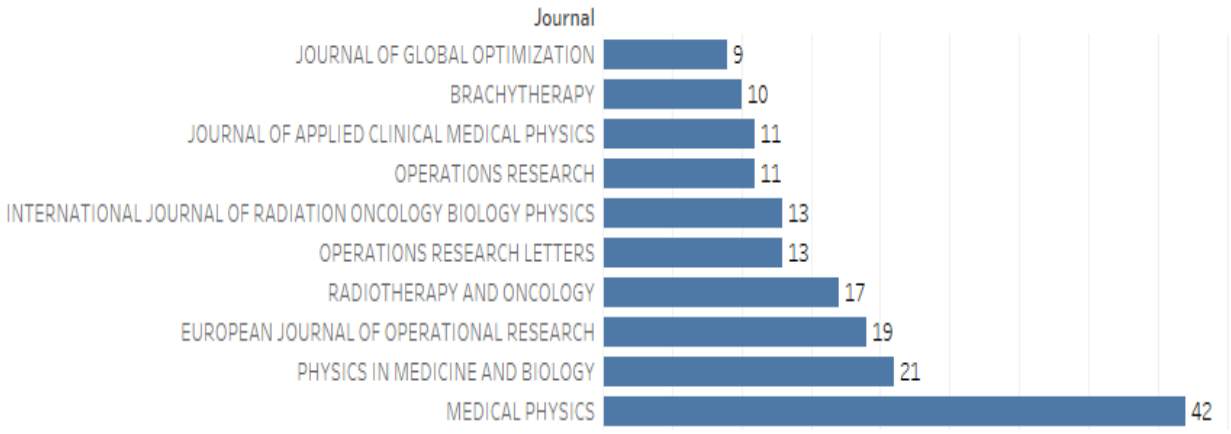

Figure 4: Publications from the top 10 highest-publishing journals.

Most Widely Cited Articles in Inverse Optimization

In Table 3, the top 5 most-cited articles from the period of 2010-2020 are presented, ranked by citation count according to data sourced from WoS (For the table of the top 30 most cited articles, see Table 6 Appendix B). The table includes detailed entries with authors' full names, journal names and keywords. Notably, the foremost paper in the field of inverse optimization by Ahuja and Orlin, which established and formalized the concept of inverse optimization. This seminal work has been pivotal in the field, reflected by its citation count of 290. The second-most cited article, a review written by Heuberger, has accrued over 200 citations, denoting significant recognition and impact. The articles appear to cover a range of topics, most notably focusing on optimization in different contexts, including combinatorial optimization and optimization in relation to physics, auctions, self-assembly, physiological modeling, material science and renewable energy. The number of citations for these articles ranges from below 100 to nearly 300, indicating their impact and relevance in their respective fields.

The high citation rates of these articles suggest they address subject matters of substantial importance within their respective fields. The lead article by Ahuja and Orlin is often cited because it laid the foundational framework for inverse optimization, an area of study with wide-ranging applications in logistics, network design and resource allocation. Its prominence is attributed to its introduction of new methodologies that have since become standard in the field. Heuberger's review paper, by systematically summarizing problems, methods and results in inverse combinatorial optimization, has likely become a key reference point for researchers and practitioners alike, explaining its extensive citation count. Review papers typically garner high citations as they provide a synthesis of existing literature and offer direction for future research efforts. The subject matter of these highly cited articles deals with foundational and methodological advances in inverse optimization. Their extensive citation is likely due to the novelty, applicability and the ongoing development in the inverse optimization discipline.

The table also highlights the interdisciplinary nature of inverse optimization, showing its application from theoretical problems in operations research to practical issues in energy policy planning. The citation counts reflect the academic community's recognition of the importance of these works. The presence of high citation counts also implies that the concepts developed in these articles are foundational and continue to influence ongoing research across various disciplines. This could be particularly valuable for researchers looking for seminal works in the field or for institutions aiming to develop programs or courses based on the most impactful research.

Also, as demonstrated in Figure 5, approximately 25% of all the papers considered in this field, or around 200 papers, have received no citations. Additionally, only a limited number of articles, less than 25, have received more than 10 citations.

Figure 5 depicts a skewed distribution of citations among papers in inverse optimization, highlighting a common trend in academic research where a small fraction of work receives the majority of attention. The fact that about 25% of papers have not been cited suggests there could be a large amount of research in the field that is not being utilized or built upon by other academics. This may indicate areas where research topics are less appealing or too niche for the wider research community, or that some papers may lack the novelty or relevance required to attract citations. On the other end, the few papers that have garnered more than 10 citations represent the more influential research within inverse optimization, potentially introducing seminal concepts, innovative methodologies, or comprehensive reviews that resonate with and are used by other researchers. These highly cited articles are likely to be the ones setting the agenda and influencing the direction of future research in the field. The disparity also underscores the importance of effective dissemination and networking within the academic community, as well as the potential impact of publishing in high-visibility journals.

Table 3: Articles with the highest citations.

Article Title	Times Cited, All Databases	Author Full Names	Journal Abbreviation	Author Keywords
Inverse optimization	290	Ahuja, RK; Orlin, JB	OPER RES	
Inverse combinatorial optimization: A survey on problems, methods and results.	211	Heuberger, C	J COMB OPTIM	Inverse optimization; reverse optimization; network flow problems.
Learning physics-based motion style with nonlinear inverse optimization.	182	Liu, CK; Hertzmann, A; Popovic, Z	ACM T GRAPHIC	Character animation; motion style; physics-based animation; inverse optimization.
An inverse-optimization-based auction mechanism to support a multiattribute RFQ process.	160	Beil, DR; Wein, LM	MANAGE SCI	Inverse optimization; multiattribute auctions; mechanism design.
Inverse optimization techniques for targeted self-assembly.	150	Torquato, Salvatore	SOFT MATTER	

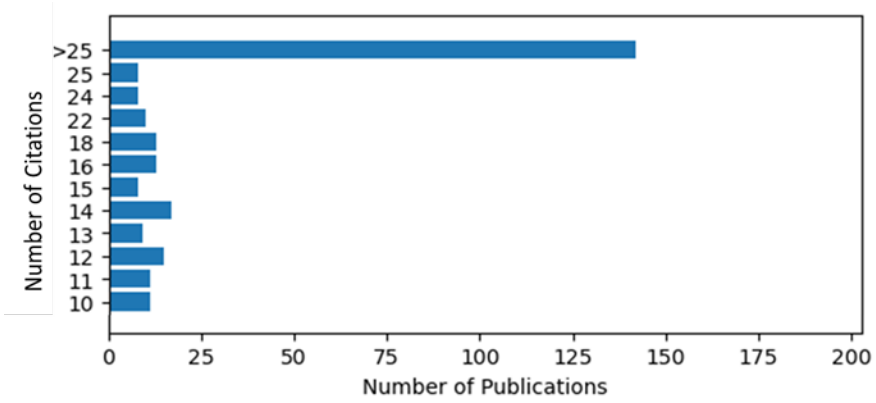

Figure 5: Citations received by papers.

Most Prolific and Influential Authors

Table 4 presents the authors with the highest total link strength, accompanied by the number of citations and documents attributed to them, as well as their affiliated institutions and the keywords frequently associated with inverse optimization in their work (For the table of the top 30 the highest overall link strength, see Appendix C.) As depicted in Table 4 and Table 7 (Appendix C), Latash, Mark L. stands out as the author with the highest number of citations. Furthermore, Alizdeh, Behrooz has the largest number of publications on this topic, with 22 documents. Additionally, Chan, Timothy C. Y., who is widely recognized as a prominent researcher in this field, holds the third place in terms of citations.

Table 4 and Table 7 reveal a landscape where a diverse group of authors contributes to the field of inverse optimization, each with varying degrees of influence and specialization. Alizadeh, Behrooz emerges as the most prolific author, suggesting a significant impact on the breadth of the field with the highest number of publications. Latash, Mark L (254). and Chan, Timothy C. Y. (249) stand out for their high citation counts, indicating that their work resonates strongly with the research community and likely drives the field's progression (See Table 6 Appendix B.). The varying total link strengths point to different degrees of collaborative engagement and influence within the research network.

As it is shown in Tables 2 and 4, the institutions of highly cited authors may not match the highly cited institutions. It may have different reasons, some of them described below.

Table 4: Authors with the highest overall link strength.

Author	Documents	Citations	Citations/ Documents	Total link strength	Institution	Frequent Keywords within Inverse Optimization
Alizadeh, Behrooz	22	239	10.86	28	Sahand University of Technology	Inverse Optimization; Combinatorial Optimization; Location Problem; Time Complexity; p-Median Location; Median Location; Facility Location; Obnoxious Center Location.
Baroughi, Fahimeh	14	46	3.29	22	Sahand University of Technology	Inverse Optimization; Combinatorial Optimization; Location Problem; p-Median; Time Complexity; Median Location; Facility Location Problem.
Afrashteh, Esmaeil	8	42	5.25	16	Sahand University of Technology	Inverse Optimization; Combinatorial Optimization; Time Complexity; Median Location; p-Median Location; Facility Location Problem.
Latash, Mark L.	13	254	19.54	16	Pennsylvania State University	Inverse Optimization; Synergy; Finger; Force; Redundancy; Prehension Synergy; Optimization; Uncontrolled Manifold Hypothesis.
Zatsiorsky, Vladimir M.	12	235	19.58	15	Pennsylvania State University	Inverse Optimization; Optimization; Grasping; Synergy; Finger; Force; Redundancy; Uniqueness Theorem; Principal Component Analysis.

Mobility of Scholars

Authors may change institutions and their citations often continue to accrue after they have moved. Their new institutions wouldn't immediately receive the credit for the citations.

Collaborations

Highly cited papers may be the result of collaborations among researchers from multiple institutions.

Aggregated Data

Citation counts for institutions may aggregate the citations of all researchers currently affiliated, regardless of where the work was done, while an author's citations follow them throughout their career.

Field-Specific Practices

Some fields might be more collaborative or have different practices in citing and authorship that could impact these metrics.

Time Lags in Reporting

There could be delays in updating citation databases with current institutional affiliations.

Institutional Support

Some institutions might have the resources to support highly cited research but may not have the most highly cited individuals if their work is spread across a wide array of researchers.

Additionally, as shown in Table 4 and Figure 3, there is a notable observation that authors who have received a high number of citations in this field do not always align with the use of highly cited keywords. This phenomenon can be attributed to several factors, which are explored further below:

Broad Influence

Authors might be cited for a broad range of work. While their most influential papers may include certain keywords, their overall citation count could be bolstered by a wider variety of publications that span different topics.

Pioneering Work

Highly cited authors may have conducted pioneering work that established new directions for a field. Such work might precede the establishment of the terms that later become highly cited keywords.

Evolving Vocabulary

The vocabulary and popular keywords of a field can evolve over time. Authors may have been highly cited for work using

terminology that was current at the time but has since been superseded or refined.

Citation Context

Citations can occur for various reasons, including criticism, comparison, or historical context, not just for endorsing or building upon the cited work. Therefore, the presence of citations does not always correlate with keyword alignment.

Keyword Generality

Highly cited keywords may be too general or broad and highly cited authors may have focused on more specific or niche topics that do not align with these broad terms.

Collaborative Works

Some authors may have a high citation count due to collaborative projects. The specific keywords might be more closely associated with their co-authors' direct contributions.

In summary, citation frequency is a complex metric influenced by various factors, including but not limited to the direct content of the publication as signified by its keywords.

DISCUSSION

This paper's bibliometric analysis of inverse optimization provides a comprehensive overview of the field's development, key contributors and thematic trends. The results are examined analytically, focusing on notable aspects and their broader implications.

Emphasis on Healthcare Applications

Inverse optimization has shown a significant emphasis on healthcare, especially in cancer treatment methods like radiation therapy and brachytherapy. This trend underscores the field's practical impact in life-saving treatments. The recurring use of keywords such as "radiation therapy," "brachytherapy," "cancer," and "IMRT" highlights this focus, illustrating the intersection of inverse optimization with critical healthcare applications. This confluence underscores the field's theoretical value and tangible benefits in enhancing treatment quality and planning in oncology.

Geographic and Institutional Influence

The analysis reveals a geographical concentration of research activity, predominantly in the USA, followed by China and Canada. Key institutions like the Massachusetts Institute of Technology (MIT) and the University of Toronto are notable for their substantial contributions, suggesting a strong support system within these academic environments. Interestingly, countries like Austria, Spain and England, despite their lower publication volume, demonstrate high impact, as evidenced by their citations per document ratio. This indicates specialized, high-quality research outputs from these nations.

Trends in Publication and Research Focus

There has been a noticeable decline in publication frequency post-2018, with a slight increase observed in 2021. This pattern is likely attributable to the impacts of the COVID-19 pandemic, reflecting the challenges encountered by the research community during global health crises. Additionally, the presence of a substantial proportion of uncited papers indicates potential underexplored areas within the field. In contrast, a small set of highly cited papers highlights influential research, shaping the direction of inverse optimization.

Interdisciplinary Nature and Journal Influence

Publications in prominent journals such as "Medical Physics," "Physics in Medicine and Biology," and "European Journal of Operational Research" reflect the interdisciplinary character of inverse optimization. The melding of operational research principles with medical applications in these journals signifies a directed effort towards developing impactful healthcare solutions, maintaining scientific integrity and rigor.

Collaborative and Individual Contributions

The field's diversity in authorship, with varying degrees of influence and expertise, illustrates a collaborative nature within inverse optimization. Researchers like Alizadeh, Behrooz and Chan, Timothy C. Y. are noteworthy for their significant contributions to advancing the field. Furthermore, the high citation rates per document for authors such as Burkard, Rainer E., Wang, Lizhi and Park, Jaebum emphasize the quality and relevance of their work.

CONCLUSION

In this study, we have presented a bibliometric overview of the field of inverse optimization. Our findings serve as a useful guide for researchers seeking to identify influential works, including key journals, papers and authors in this domain. While our results generally align with the academic community's perceptions, some discrepancies in numerical data are present. Notably, there has been a slowdown in the number of publications in this field, except for a spike in 2021. This may be attributed to the challenges in making novel contributions within this complex area. Additionally, the decline in non-COVID-19 research during the pandemic[27] likely influenced this trend. Most publications in inverse optimization are linked to cancer treatment or theoretical developments, with the United States and MIT being particularly prominent and influential.

It is important to acknowledge the limitations of this study, particularly regarding the quality of the journals and papers analyzed. Some authors, though having fewer publications, may contribute works of higher quality. Thus, considering citation counts and expert opinions about these authors would yield a more comprehensive assessment. This consideration is equally

relevant to other elements such as institutions, countries and articles. The varying prestige and impact of different journals can influence the outcomes of our analysis. Expanding the scope to include other databases like Scopus could offer a more rounded view. For future research, integrating machine learning algorithms might enhance the depth and breadth of the analysis.

DATA AVAILABILITY STATEMENT

The datasets analysed during the current study are available in the "inverse_optimization.csv" repository, https://drive.google.com/file/d/17RD8hp-TwL7TiJIdn-90ODRksIcqfzA0/view?usp=drive_link

CONFLICT OF INTEREST

The authors declare that there is no conflict of interest.

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Appendix A

Table 5: Institutions with the highest overall link strength.

Organization	Documents	Citations	Citations/ Documents	Total link strength	Authors Keywords
University of Toronto	21	524	24.95	11	Inverse optimization Linear Programming Treatment Planning Machine Learning Knowledge-Based Planning Radiation Therapy Clinical Pathway Concordance Cancer Therapy (e.g., colon cancer, breast cancer, prostate cancer) Multiagent System Network Analysis.
Massachusetts General Hospital	7	50	7.14	10	Proton Therapy Pencil Beam Scanning (PBS) Intensity Modulated Proton Therapy (IMPT) Alternating Direction Method of Multipliers (ADMM) Knowledge-Based Planning Optimization Inverse Optimization.
Emory University	11	77	7.00	9	Proton Therapy Pencil Beam Scanning (PBS) Intensity Modulated Proton Therapy (IMPT) Alternating Direction Method of Multipliers (ADMM) Knowledge-Based Planning Optimization Inverse Optimization.
Harvard Medical School	6	52	8.67	9	Proton Therapy Pencil Beam Scanning (PBS) Intensity Modulated Proton Therapy (IMPT) Alternating Direction Method of Multipliers (ADMM) Minimum Monitor Unit (MU) Knowledge-Based Planning Inverse Treatment Planning Brachytherapy.

Organization	Documents	Citations	Citations/ Documents	Total link strength	Authors Keywords
MIT	15	683	45.53	8	Inverse Optimization Robust Optimization Linear Programming Market Elements (Electricity Pool, Marginal Prices, Market Clearing) Mechanism Design Auctions (Multiattribute Auctions) Equilibrium Estimation (Nonparametric Estimation, Utility Estimation) Traffic Assignment Flow Problems (Maximum-Flow Problem, Minimum-Cut Problem, Minimum-Cost Flow Problem, Minimum Mean-Cycle Problem) Combinatorial Optimization Computational Complexity Mixed-Integer Programming.
Southeast University	7	37	5.29	8	Inverse Optimization Spanning Tree Problem (specifically, Max Plus Sum Spanning Tree Problem and Inverse Max Plus Sum Spanning Tree Problem) Linear Programming Hamming Distance Norms (l(0) norm, l(infinity) norm) Binary Search Algorithm Design (Column Generation Algorithm, Binary Search, Strongly Polynomial Time Algorithm).
University of Florida	9	453	50.33	8	Inverse Optimization Parameter Estimation Spanning Tree Problem (Max Plus Sum Spanning Tree Problem and Inverse Max Plus Sum Spanning Tree Problem) Linear Programming Optimization Problems (Maximum-Flow Problem, Minimum-Cut Problem, Minimum-Cost Flow Problem, Minimum Mean-Cycle Problem, Minimax Problems) Norms (l(0) norm, l(infinity) norm) Algorithms (Multilevel Coordinate Search, Column Generation Algorithm, Binary Search, Strongly Polynomial Time Algorithm, Discrete Type Newton Method).

Organization	Documents	Citations	Citations/ Documents	Total link strength	Authors Keywords
Graz University of Technology	10	478	47.80	7	Inverse Optimization Location Problem Combinatorial Optimization Complexity Analysis Network Center Location Median Problems (1-median, p-median, Ordered median, k-centrum) Network Flow Problems Facility Location Problem.
Hohai University	5	67	13.40	7	Inverse Optimization Spanning Tree Problem (Minimum Spanning Tree, Max Plus Sum Spanning Tree Problem) Hamming Distance Linear Programming Binary Search Dual Theory Column Generation Algorithm.
Sahand University of Technology	24	282	11.75	7	Inverse Optimization Combinatorial Optimization Location Problem (including Facility Location Problem, Median Location Problem, Center Location Problem) p-Median Time Complexity Network Center Location Optimization Algorithms (like Firefly Algorithm, Hybrid Algorithm, Particle Swarm Optimization Algorithm, Modified Bat Algorithm) Tree Networks (including Tree Graphs, AVL-Search Tree, Red-Black Search Tree).
Can Tho University	11	40	3.64	6	Inverse Optimization Location Problem Tree Chebyshev Norm Hamming Distance Median Problem (including variations like Group Median, 1-Center Problem, p-Maxian) Complexity Convexity Parameterization Robustness Uncertainty Combinatorial Optimization Knapsack Problem (Continuous Knapsack Problem) Ordered Median Function.

Organization	Documents	Citations	Citations/ Documents	Total link strength	Authors Keywords
University of Houston	7	60	8.57	6	Inverse Optimization Linear Programming Robust Optimization Cancer Therapy Objective Selection Feature Selection Multiobjective Optimization Dietary Recommendation Model Estimation Convex Programming Online Learning Clustering Polyhedral Theory.
City University of Hong Kong	10	278	27.80	5	Inverse Optimization Optimization Method/Effectiveness (including Particle Swarm Optimization, Revised Simplex Method, Column Generation Method, Ellipsoid Method) Location Problem Complexity Networks and Graphs Minimum Spanning Tree Assignment Problem Thermal RC Model Thermoelectric Generator/Material Semi-definite Quadratic Programming Special Linear Program Polynomially Solvable Problems Bottleneck Problem Parametric Linear Programming.
Concordia University	6	40	6.67	5	Optimization (including Robust Optimization, Discrete Optimization, Integer Optimization) Linear Programming Cutting Plane Algorithm Imperfect Observations Deep Learning Interior Point Dietary Recommendation Goodness of Fit Model Estimation Utility Elicitation Assortment Optimization Machine Learning Demand Prediction Multiattribute Decision Making.

Organization	Documents	Citations	Citations/ Documents	Total link strength	Authors Keywords
Ton Duc Thang University	5	19	3.80	5	Inverse Optimization Location Problem Tree/Trees Chebyshev (norm) Median Problem (including Group Median, 1-center) Hamming Distance Complexity Parameterization Interval Graph Ordered Median Function Anti-k-centrum Cycle.
Princess Margaret Cancer Centre	5	137	27.40	5	nverse Optimization Treatment Planning Knowledge-Based Planning Machine Learning Radiation Therapy IMRT (Intensity-Modulated Radiation Therapy) Prostate Cancer Treatment Automated Planning Objective Function Weights Multicriteria Open Data.
Shanghai Jiao Tong University	7	83	11.86	4	Optimization (accompanying various contexts such as knowledge-based planning, market strategy and portfolio optimization) Planning (including knowledge-based planning, planning strategy) Learning (On-line learning, supervised learning) Radiotherapy Automated Planning Open Data Gaussian Processes Big Data Metamaterials Design (Design Automation, Design Optimization, Design Representation).

Organization	Documents	Citations	Citations/ Documents	Total link strength	Authors Keywords
University of California, Berkeley	9	137	15.22	4	Optimization (including Nonlinear Optimization and Direct Aperture Optimization) Estimation (accompanying various contexts like structural estimation and parameter optimization) Machine Learning Statistical Learning Semiparametric Algorithm Game Theory Mixed Integer Linear Programming (MILP) Radiotherapy (including terms like 4pi radiotherapy and knowledge-based planning) Goal Programming Vehicle Routing Problem Freight Forecast Network Assignment Contract Design Healthcare Policy Set-valued Functions Complexity Shortest Path Parametric Search Adaptive User Interfaces Radiosurgery Soil Hydraulic Conductivity Public Policy Decision Analysis.
Northwestern University	7	48	6.86	3	Inverse Optimization Bayesian Optimization Gaussian Processes Supervised Learning Big Data Metamaterials Design Optimization Deep Learning Data-Driven Modeling.

Organization	Documents	Citations	Citations/ Documents	Total link strength	Authors Keywords
Boston University	9	185	20.56	2	Equilibrium (including Wardrop Equilibrium and Market Equilibrium) Learning Linear Programming Metabolic Networks Nonparametric Estimation Demand Response Pricing Traffic Assignment Computational Biology Kernel Methods Variational Inequalities Flux Balance Analysis Genome-Scale Stoichiometric Models.
Chinese Academy of Sciences	9	92	10.22	2	Optimization (including Inverse Optimization, Partial Inverse Optimization, Nonlinear Optimization) Location Problem Complexity Radiotherapy (including IMRT, VMAT and references to specific types such as cervical cancer radiation therapy) Networks and Graphs (related to path, cycle, assignment, spanning tree and graph-based methods in computational lithography) Planning (including knowledge-based planning, planning strategy and automated planning) Deep Learning (applied to cervical cancer and computational lithography) Computational Lithography Animal Trampling (related to model, soil properties, evapotranspiration) Assignment Problem Polynomially Solvable Problems Open Data.

Organization	Documents	Citations	Citations/ Documents	Total link strength	Authors Keywords
Dalian University of Technology	20	112	5.60	2	Linear Programming Quadratic Programming Newton Method (including Smoothing Newton Method, Inexact Newton Method, Damped Semismooth Newton Method) Perturbation Approach Complementarity Constraints Vector Norms (specifically l(1) vector norm) Semidefinite Quadratic Programming Second-Order Cone Quadratic Programming Augmented Lagrangian Method Rate of Convergence Human Resource Allocation Supply Chain Damage Identification.
University of Amsterdam	5	94	18.80	2	Inverse optimization Linear programming Quadratic programming Newton method Perturbation approach Smoothing Newton method Semidefinite quadratic programming Second-order cone quadratic programming Augmented Lagrangian method Convergence rate.
University of California, Los Angeles	6	78	13.00	2	Combinatorial optimization Inverse optimization Minimum spanning tree Shortest path Linear assignment Prostate neoplasms Brachytherapy Inverse planning IPSA (Inverse Planning Simulated Annealing) HIPO (Hybrid Inverse Planning Optimization).
University of Pittsburgh	6	94	15.67	2	Inverse optimization Infinite-dimensional optimization Duality theory Character animation Motion style Physics-based animation Linear programming Optimization Game theory Smart building energy efficiency.

Organization	Documents	Citations	Citations/ Documents	Total link strength	Authors Keywords
University of Washington	9	230	25.56	2	Inverse optimization Infinite-dimensional optimization Duality theory Character animation Motion style Physics-based animation Game theory Smart building energy efficiency Linear programming Optimization.
Zhejiang University	5	83	16.60	2	Inverse optimization Optimization Multi-objective Hamming distance Minimum spanning tree Min-max spanning tree Inverse problem Strongly polynomial algorithms Minimum cut k-space Fourier transform.
Delft University of Technology (TU Delft)	5	73	14.60	1	Supervised learning Aerospace electronics Training Linear programming Cost function Symmetric matrices Least mean squares methods Learning-based control Inverse optimization Convex reformulation.
Duke University	5	18	3.60	1	Image processing Inelastic neutron scattering Hierarchical optimization Mathematical regularization SRS (Stereotactic Radiosurgery) Single-isocenter VMAT (Volumetric Modulated Arc Therapy) DCA (Dynamic Conformal Arc) Artificial intelligence Machine learning.

Appendix B

Table 6: Articles with the highest citations.

Article Title	Times Cited, All Databases	Author Full Names	Journal Abbreviation	Author Keywords
Inverse optimization	290	Ahuja, RK; Orlin, JB	OPER RES	
Inverse combinatorial optimization: A survey on problems, methods and results.	211	Heuberger, C	J COMB OPTIM	Inverse optimization; reverse optimization; network flow problems
Learning physics-based motion style with nonlinear inverse optimization.	182	Liu, CK; Hertzmann, A; Popovic, Z	ACM T GRAPHIC	Character animation; motion style; physics-based animation; inverse optimization
An inverse-optimization-based auction mechanism to support a multiattribute RFQ process.	160	Beil, DR; Wein, LM	MANAGE SCI	Inverse optimization; multiattribute auctions; mechanism design.
Inverse optimization techniques for targeted self-assembly.	150	Torquato, Salvatore	SOFT MATTER	
The complexity analysis of the inverse center location problem.	92	Cai, MC; Yang, XG; Zhang, JZ	J GLOBAL OPTIM	Complexity; location problem; networks and graphs; satisfiability problem.
Inverse optimization: Functional and physiological considerations related to the force-sharing problem.	91	Tsirakos, D; Baltzopoulos, V; Bartlett, R	CRIT REV BIOMED ENG	Mathematical modeling; optimization; force sharing; indeterminant system.
An inverse optimization strategy to determine single crystal mechanical behavior from polycrystal tests: Application to AZ31 Mg alloy.	89	Herrera-Solaz, V.; LLorca, J.; Dogan, E.; Karaman, I.; Segurado, J.	INT J PLASTICITY	Crystal plasticity; Polycrystal homogenization; Inverse analysis; Levenberg-Marquardt; AZ31 Mg alloy.
Designing effective and efficient incentive policies for renewable energy in generation expansion planning.	89	Zhou, Ying; Wang, Lizhi; McCalley, James D.	APPL ENERG	Generation expansion planning; Renewable energy; Incentive policy; Mandatory policy; Bilevel optimization; Inverse optimization.
An improved CSS for damage detection of truss structures using changes in natural frequencies and mode shapes.	87	Kaveh, A.; Zolghadr, A.	ADV ENG SOFTW	Damage identification; Improved charged system search; Truss structures; Frequencies; Mode shapes; Inverse problem.
Data-driven estimation in equilibrium using inverse optimization.	81	Bertsimas, Dimitris; Gupta, Vishal; Paschalidis, Ioannis Ch.	MATH PROGRAM	Equilibrium; Nonparametric estimation; Utility estimation; Traffic assignment.
Calibration of Richards' and convection-dispersion equations to field-scale water flow and solute transport under rainfall conditions.	79	Jacques, D; Simunek, J; Timmerman, A; Feyen, J	J HYDROL	Parameter optimization; numerical modeling; unsaturated zone; water and solute transport.
High-dose simultaneously integrated breast boost using intensity-modulated radiotherapy and inverse optimization.	74	Hurkmans, Coen W.; Meijer, Gert J.; van Vliet-Vroegindeweij, Corine; van der Sangen, Maurice J.; Cassee, Jorien	INT J RADIAT ONCOL	Breast cancer; intensity-modulated radiotherapy; simultaneous integrated boost; inverse planning.

Article Title	Times Cited, All Databases	Author Full Names	Journal Abbreviation	Author Keywords
Practical Implementation of Attitude-Control Algorithms for an Underactuated Satellite.	74	Horri, Nadjim M.; Palmer, Phil	J GUID CONTROL DYNAM	
HyperReconNet: Joint Coded Aperture Optimization and Image Reconstruction for Compressive Hyperspectral Imaging.	67	Wang, Lizhi; Zhang, Tao; Fu, Ying; Huang, Hua	IEEE T IMAGE PROCESS	Compressive hyperspectral imaging; convolution neural network; coded aperture optimization; hyperspectral image reconstruction.
Learning Layouts for Single-Page Graphic Designs.	65	O'Donovan, Peter; Agarwala, Aseem; Hertzmann, Aaron	IEEE T VIS COMPUT GR	Graphic design; layout; modeling; learning; crowdsourcing; nonlinear inverse optimization.
Modeling grazing effects on coupled water and heat fluxes in Inner Mongolia grassland.	65	Zhao, Ying; Peth, Stephan; Horn, Rainer; Kruemmelbein, Julia; Ketzer, Bettina; Gao, Yingzhi; Doerner, Jose; Bernhofer, Christian; Peng, Xinhua	SOIL TILL RES	Model; Animal trampling; Soil properties; Water and heat fluxes; Evapotranspiration; Inner Mongolia grassland.
Inverse conic programming with applications.	63	Iyengar, G; Kang, WM	OPER RES LETT	Inverse optimization; conic programming; portfolio optimization; utility function identification.
A stochastic model updating method for parameter variability quantification based on response surface models and Monte Carlo simulation.	63	Fang, Sheng-En; Ren, Wei-Xin; Perera, Ricardo	MECH SYST SIGNAL PR	Stochastic model updating; Parameter variability; Analysis of variance; Response surface models; Monte Carlo simulation.
Modeling cyclic deformation of inconel 718 superalloy by means of crystal plasticity and computational homogenization.	63	Cruzado, A.; LLorca, J.; Segurado, J.	INT J SOLIDS STRUCT	Crystal plasticity; Computational homogenization; Cyclic behavior; Mean stress relaxation; Inconel 718 superalloy.
Inverse data envelopment analysis model to preserve relative efficiency values: The case of variable returns to scale.	60	Lertworasirikul, Saowanee; Charnsethikul, Peerayuth; Fang, Shu-Cherng	COMPUT IND ENG	Data envelopment analysis; Inverse optimization; Efficiency; Performance analysis; Resource allocation.
A multipopulation genetic algorithm to solve the inverse problem in hydrogeology.	59	Karpouzos, DK; Delay, F; Katsifarakis, KL; de Marsily, G	WATER RESOUR RES	
Determination of parameters for bimodal hydraulic functions by inverse modeling.	59	Zurmuhl, T; Durner, W	SOIL SCI SOC AM J	
Inverse Optimization: A New Perspective on the Black-Litterman Model.	57	Bertsimas, Dimitris; Gupta, Vishal; Paschalidis, Ioannis Ch.	OPER RES	

Article Title	Times Cited, All Databases	Author Full Names	Journal Abbreviation	Author Keywords
Generalized Inverse Multiobjective Optimization with Application to Cancer Therapy.	57	Chan, Timothy C. Y.; Craig, Tim; Lee, Taewoo; Sharpe, Michael B.	OPER RES	
Effect of rare earth additions on the critical resolved shear stresses of magnesium alloys.	56	Herrera-Solaz, V.; Hidalgo-Manrique, R.; Perez-Prado, M. T.; Letzig, D.; Llorca, J.; Segurado, J.	MATER LETT	Critical resolved shear stresses; Crystal plasticity; Rare earths; Magnesium alloys; Inverse optimization.
Tri-criterion inverse portfolio optimization with application to socially responsible mutual funds.	56	Utz, Sebastian; Wimmer, Maximilian; Hirschberger, Markus; Steuer, Ralph E.	EUR J OPER RES	Socially responsible investing; Inverse optimization; Portfolio selection; Multiple criteria optimization; Nondominated surfaces; Multiple criteria decision making
Inverse planning approach for 3-D MRI-based pulse-dose rate intracavitary brachytherapy in cervix cancer.	56	Chajon, Enrique; Dumas, Isabelle; Touleimat, Mahmoud; Magne, Nicolas; Coulot, Jeremy; Verstraet, Rodolfe; Lefkopoulos, Dimitri; Haie-Meder, Christine.	INT J RADIAT ONCOL	Brachytherapy; cervical cancer; inverse planning; pulsed-dose rate.
Network Design and Capacity Exchange for Liner Alliances with Fixed and Variable Container Demands.	54	Zheng, Jianfeng; Gao, Ziyou; Yang, Dong; Sun, Zhuo	TRANSPORT SCI	Liner alliances; mixed-integer linear programming; inverse optimization.
Field quantification of wetting-drying cycles to predict temporal changes of soil pore size distribution.	54	Bodner, G.; Scholl, P.; Kaul, H. -P.	SOIL TILL RES	Wetting-drying cycles; Soil pore size distribution; Spectral analysis; Temporal variability; Tension infiltrometer.

Appendix C

Table 7: Authors with the highest overall link strength.

Author	Documents	Citations	Citations/ Documents	Total link strength	Institution	Frequent Keywords within Inverse Optimization
Alizadeh, Behrooz	22	239	10.86	28	Sahand University of Technology	Inverse Optimization Combinatorial Optimization Location Problem Time Complexity p-Median Location Median Location Facility Location Obnoxious Center Location.
Baroughi, Fahimeh	14	46	3.29	22	Sahand University of Technology	Inverse Optimization Combinatorial Optimization Location Problem p-Median Time Complexity Median Location Facility Location Problem.
Afrashteh, Esmaeil	8	42	5.25	16	Sahand University of Technology	Inverse Optimization Combinatorial Optimization Time Complexity Median Location p-Median Location Facility Location Problem.
Latash, Mark L.	13	254	19.54	16	Pennsylvania State University	Inverse Optimization Synergy Finger Force Redundancy Prehension Synergy Optimization Uncontrolled Manifold Hypothesis.
Zatsiorsky, Vladimir M.	12	235	19.58	15	Pennsylvania State University	Inverse Optimization Optimization Grasping Synergy (often paired with 'prehension') Finger Force Redundancy Uniqueness Theorem Principal Component Analysis.

Author	Documents	Citations	Citations/ Documents	Total link strength	Institution	Frequent Keywords within Inverse Optimization
Zhang, Liwei	12	76	6.33	12	BJNU-HKBU United International College	Inverse Optimization Quadratic Programming Newton Method (including variations like Inexact Newton Method, Smoothing Newton Method, Damped Newton Method) Perturbation Approach Convergence Rate/Rate of Convergence Smoothing Function l(1) Vector Norm Semidefinite Quadratic Programming Augmented Lagrangian Method Second-Order Cone Programming.
Chan, Timothy C. Y.	14	249	17.79	11	University of Toronto	Inverse Optimization Linear Programming Treatment Planning Machine Learning Knowledge-Based Planning Optimization.
Lee, Taewoo	10	192	19.20	11	University of Pittsburgh	Inverse Optimization Linear Programming Multiobjective Optimization Cancer Therapy (with a specific mention of prostate cancer treatment and OR in cancer therapy) Dietary Recommendation Model Estimation Robust Optimization Feature Selection Greedy Algorithm Machine Learning.
Zhang, Jianzhong	11	163	14.82	11	BJNU-HKBU United International College	Inverse Optimization Linear Programming Quadratic Programming Perturbation Approach Newton Method (and its variations like Inexact Newton Method, Smoothing Newton Method) Rate of Convergence KKT Conditions Polynomially Solvable Problems Assignment Problem Semi-definite Quadratic Programming Augmented Lagrangian Method.

Author	Documents	Citations	Citations/ Documents	Total link strength	Institution	Frequent Keywords within Inverse Optimization
Xiao, Xiantao	6	46	7.67	10	Dalian University of Technology	Inverse Optimization Quadratic Programming Newton Method (including specific methods like Inexact Newton Method, Smoothing Newton Method) Perturbation Approach Convergence Rate Smoothing Function Linear Programming Augmented Lagrangian Method.
Park, Jaebum	5	140	28.00	9	Pennsylvania State University	Inverse Optimization Finger Force Redundancy Synergy Uncontrolled Manifold Hypothesis ANIO Approach.
Hajimirza, Shima	12	61	5.08	8	University of Texas at Austin	Thin Film Solar Cells Inverse Optimization Numerical Optimization Sensitivity Analysis Spectral Absorptivity Fabrication Error.
Howell, John R.	8	27	3.38	8	University of Texas at Austin	Thin Film Solar Cells Inverse Optimization Sensitivity Analysis Fabrication Error Numerical Simulation Light Trapping Thin Film Deposition Monte Carlo Simulation.
Kien Trung Nguyen	15	144	9.60	8	Cantho University	Inverse Optimization Location Problem Tree Median Problem/1-Center Problem/p-Maxian Combinatorial Optimization Complexity Convex/Convexity Parameterization Chebyshev.

Author	Documents	Citations	Citations/ Documents	Total link strength	Institution	Frequent Keywords within Inverse Optimization
Sharpe, Michael B.	5	157	31.40	8	University of Toronto	Inverse Optimization Treatment Planning Automated Planning Radiation Therapy Machine Learning Multicriteria IMRT (Intensity-Modulated Radiation Therapy) Prostate Cancer Treatment.
Burkard, Rainer E.	7	251	35.86	6	Graz University of Technology	Location Problem Inverse Optimization Combinatorial Optimization 1-median p-median Network Center Location Facility Location Problem Complexity/Complexity Analysis/ Computational Complexity.
Das, Ranjan	12	210	17.50	5	Indian Institute of Technology Ropar	Inverse Optimization Genetic Algorithm Sensitivity Analysis Constrained Optimization Exergy Destruction Feedback and Control Heat Transfer Analysis Energy Systems Analysis.
Gao, Hao	7	49	7.00	5	University of Kansas	Proton Therapy Intensity Modulated Proton Therapy (IMPT) Pencil Beam Scanning (PBS) Alternating Direction Method of Multipliers (ADMM) Treatment Planning Inverse Optimization Minimum Monitor Unit (MU) Hybrid Proton-Photon Optimization.
Lin, Yuting	5	31	6.20	5	University of Kansas	Proton Therapy Pencil Beam Scanning (PBS) Intensity Modulated Proton Therapy (IMPT) Alternating Direction Method of Multipliers (ADMM) Treatment Planning Inverse Optimization.
Mihaylov, I.	7	0	0.00	5	University of Miami	Optimization IMRT (Intensity Modulated Radiation Therapy) Cancer.